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TECHNICAL MEMORANDUM

WOBBLE DAMPING OF SPINNING SATELLITES USING CMGS WITH APPLICATION TO A SKYLAB B ARTIFICIAL-G

Bellcomm

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ABSTRACT

Wobble damping using CMGs is investigated for possible application on a Skylab artificial gravity experiment. Control torques provided by CMGs are assumed proportional to spacecraft angular velocity components perpendicular to its spin axis. This control law is found to damp wobble but results in accumulated CMG angular momentum. A tradeoff is shown to exist between the wobble decay time and CMG momentum accumulation.

Venting torques and small misalignments of spacecraft control axes from principal axes are both found to result in a secular component of CMG angular momentum. While the momentum buildup rate associated with venting torques is negligible, the buildup rate associated with control axis misalignments is not; CMG momentum saturation can occur within an hour. To alleviate this problem, continual realignment of control axes along principal axes may be needed. For purposes of continual realignments, a method is given for locating principal axes using measurements of the spacecraft angular velocity. Also helpful is switching off the control when wobble velocities reach an acceptably low level.

Torques due to gravity gradient, crew motion, and venting were found to produce undamped wobble motion insufficient to be detrimental to crew comfort (based on current estimates for limits on man's physiological tolerances and adaptability). Consequently, wobble damping may not be necessary, particularly in view of damping phenomena inherent in the Skylab vehicle.

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ABSTRACT ONLY

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SUBJECT: Wobble Damping of Spinning Satellites Using CMGs With Application to a Skylab-B Artificial-G

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FROM: R. A. Wenglarz

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TECHNICAL MEMORANDUM

INTRODUCTION

As manned spacecraft have become more sophisticated and as mission durations have become more prolonged, serious thought has been given to providing crews with an environment approaching that on earth. A step in this direction is artificial gravity obtained by spinning of the spacecraft. Such an artificial gravity option is being considered for the proposed second Skylab.

For the purposes of crew comfort, it is desirable for the spacecraft angular velocity vector to be fixed in inertial space and hence in the spacecraft as well. Any non-constant components of angular velocity in a spacecraft frame are termed wobble velocities and elimination of these is the objective of wobble damping.

Pringle[1] has rigorously shown that a perturbed spinning body subjected to internal damping experiences asymptotically stable motion if and only if the spin axis is the axis of maximum moment of inertia. This implies not only that a spacecraft must be spun about this axis but also that wobble damping can be accomplished by providing control torques which appear as damping terms in the spacecraft equations of motion. Control torques proportional to angular velocity components in the directions of axes associated with the two spacecraft smaller principal moments of inertia can be shown to have this form.

The work of Appendix A investigates wobble motion and CMG requirements for a spinning satellite. The CMGs provide control torques proportional to angular velocity components along the control axes, which are considered slightly misaligned from satellite principal axes. In practice, uncertainties in construction, structural deformation, and relocation of crew and equipment within a spacecraft prevent precise knowledge of the location of principal axes.

For eventual application to venting, the effect of small, constant, body-fixed torques on a spacecraft are considered in Appendix B. Both Appendices A and B neglect gravity gradient, aerodynamic, and other environmental torques.

This memorandum is arranged as follows: The motion of Skylab is determined for a worst case crew motion and a method is given for continual realignment of control axes along principal axes. Then, assuming this method results in negligibly small misalignments, CMG requirements are determined for the worst case crew motion and for constant venting torques.

WOBBLE DAMPING FOR SKYLAB

A Skylab configuration modified with masses on booms and spinning about its axis of maximum moment of inertia (hereafter designated the X_3 axis) is being studied[2] for artificial gravity experiments. Principal moments of inertia for this configuration are

$$I_1 = 0.929\times10^6 \qquad I_2 = 4.405\times10^6 \qquad I_3 = 4.572\times10^6$$

all in slug-ft^2. Three CMGs each with 2000 ft-lb-sec spin angular momentum and each with a 120 ft-lb maximum torque limit are available for attitude control.

The CMGs are assumed to implement a control torque $\underline{T}^c$ with components proportional to angular velocities ω_1' and ω_2', about two control axes in directions of unit vectors $\underline{i}_1'$ and $\underline{i}_2'$. The constant of proportionality K is the same for both axes and $\underline{i}_1'$ and $\underline{i}_2'$ are slightly misaligned from Skylab principal axes.

$$\underline{T}^c = -K\omega_1' \, \underline{i}_1' - K\omega_2' \, \underline{i}_2'$$

One source of wobble is crew motion. A worst case crew motion is considered to result when three 200 lb astronauts move from the Skylab mass center to a position $x_1' = 33.8$ ft, $x_2' = 10$ ft, $x_3' = 10$ ft in the directions of $\underline{i}_1'$, $\underline{i}_2'$, $\underline{i}_3'$ respectively, where $\underline{i}_3'$ is perpendicular to $\underline{i}_1'$ and $\underline{i}_2'$. For the spacecraft initially spinning at a constant rate about the axis of maximum moment of inertia X_3 and for instantaneous movement of the crew, conservation of angular momentum may be used to determine that Skylab receives instantaneous angular velocity impulses

$$\omega_{10} = 8.41\times10^{-3}\omega \qquad \omega_{20} = 1.15\times10^{-2}\omega$$

MISALIGNMENT OF CONTROL AXES

The 1,2 sequence of small Euler angle rotations μ_1 and μ_2 that rotate the unit vector $\underline{i}_3'$ into the direction of X_3 are assumed

$$\mu_1 = \mu_2 = 0.707°$$

Applying the results of Appendix A, several statements can be made concerning spacecraft motion and CMG requirements for Skylab. Even for small control axis misalignments, the assumed control law damps wobble so that in the steady state Skylab is spinning about a fixed line very near its X_3 axis. Also, in the steady state, the CMGs attain a periodic component of angular momentum in a plane nearly perpendicular to X_3 and a secular component nearly in the direction of X_3. A tradeoff exists between CMG momentum accumulation and the damping time t_D, defined herein as the time for wobble to decay to 10% of the initial value. Decreasing K and consequently increasing t_D results in decreasing both the maximum magnitude of the periodic component of the CMG momentum, hereafter called ΔH_{max}, and the buildup rate of the secular momentum component, hereafter called $\Delta H_3/\Delta t$. Decrease of t_D always is at the expense of increased accumulation of both CMG angular momentum components.

These tradeoffs between t_D and momentum accumulation are demonstrated in Figs.(1)-(3). Fig.(1) gives ΔH_{max} with respect to t_D for ω=2 rpm (upper curve) and for ω=6 rpm (lower curve). For Fig.(1), points are indicated on the curves for which control torques do not exceed the designated values T_{max}. These correspond to limits on the values of K. Limiting the maximum control torque limits K for a particular set of initial conditions as shown in Eq.(1) for control torque. Figs.(2) and

(3) give $\Delta H_3/\Delta t$ in 10^4 ft-lb-sec per hour with respect to t_D for $\omega = 2$ rpm and $\omega = 6$ rpm, respectively. Considering minimum damping times corresponding to the 360 ft-lb CMG torque limit, buildup rates are 22,000 ft-lb-sec/hr for $\omega = 2$ rpm and 10,000 ft-lb-sec/hr for $\omega = 6$ rpm. Both of these saturate the capacity of the three Skylab CMGs in a fraction of an hour, a clearly unacceptable situation.

Since the secular components of angular momentum can be shown to be proportional to quadratic terms in μ_1 and μ_2 and go to zero for no misalignments of the control axes, these excessive accumulation rates can be alleviated by continual realignment of control axes to the directions of spacecraft principal axes. The problem of finding the location of principal axes can be resolved by using the two relations of Eq. (24) of Appendix A and solving these for μ_1 and μ_2 in terms of steady state components V_1' and V_2' of spacecraft angular velocities in the directions of the control axes

$$\mu_1 = -\frac{1}{\omega}[V_2' + V_1'K/\omega(I_3-I_2)] \qquad \mu_2 = \frac{1}{\omega}[V_1' - V_2'K/\omega(I_3-I_1)]$$

Since K, I_1, I_2, I_3 are known and ω, V_1', V_2' can be measured, misalignments μ_1 and μ_2 can be calculated.

Also, simply turning off control when wobble velocities reach an acceptably low level would prevent continual accumulation of components of CMG angular momentum caused by any inaccuracies in the realignment procedure. In addition, excessive CMG momentum accumulation in the X_3 direction can be transferred back to Skylab by using the CMGs to impart a torque to the spacecraft in the X_3 direction, thereby changing its spin speed slightly probably without significantly increasing its wobble.

CONTROL AXES ALIGNED

Now even though the misalignments of control axes are made extremely small and momentum accumulation in the direction of X_3 is maintained at a low level, the magnitude of the accumulated periodic component of angular momentum is of concern. It is desirable to keep ΔH_{max} associated with an individual crew motion small in comparison to CMG momentum capacity. Then, numerous crew motions would not cause CMG momentum saturation because over a large number of disturbances the resulting momentum accumulations tend to cancel each other.

Fig. (4) demonstrates the tradeoff between t_D and ΔH_{max} resulting from wobble damping of the worst case crew motion for control axis misalignments assumed negligibly small. This curve applies to Skylab for $\omega \geq 2$ rpm and shows that by increasing t_D through decreasing K, ΔH_{max} can always be reduced to an acceptable level. Increasing K decreases t_D but at the expense of increased ΔH_{max}. As indicated in Fig. (4), the maximum allowable K that insures the 360 ft-lb CMG torque limit is not exceeded by the worst case crew motion gives minimum t_D's of 0.8 minutes with 850 ft-lb-sec momentum accumulation for $\omega = 2$ rpm and 2.4 minutes with 275 ft-lb-sec momentum accumulation for $\omega = 6$ rpm. For operation at torque levels less than CMG torque capacity, lower maximum torque values are indicated on Fig. (4).

VENTING TORQUES

The analysis of Appendix B gives the effects of small constant vehicle-fixed torques on satellite motion and controller requirements for the assumed control law. By considering venting torques[3]

$$T_1 = 0.096 \qquad T_2 = 0.048 \qquad T_3 = 0.187$$

all in newton-meters, initially applied while Skylab is spinning about its axis of maximum moment of inertia X_3, assuming $K_1 = K_2 = K$ and $\mu_1 = \mu_2 = 0$, and applying the results of Appendix B, several statements can be made.

Effects of venting torques are the same in nature but much less severe in extent to effects of control axis misalignments. Skylab wobble is damped such that the spacecraft in the steady state spins about a line very near to X_3. In the steady state, the CMGs attain a periodic component of angular momentum with maximum magnitude ΔH_{max} in a plane nearly perpendicular to X_3 and a secular component ΔH_3 nearly in the direction of X_3. Fig. (5) gives ΔH_{max} for K sized at each spin speed such that the worst case crew motion would not produce torques exceeding 360 ft-lbs. Fig. (6) gives $\Delta H_3/\Delta t$ in ft-lb-sec/day with K sized for each spin speed as in Fig. (5). Compared to the 6000 ft-lb-sec capacity of three CMGs, Figs. (5) and (6) show that venting does not cause excessive CMG angular momentum accumulation.

CONCLUSION

Since wobble damping using CMGs requires alignment of control axes and other measures for preventing CMG saturation, additional computer capacity may have to be added to Skylab to implement wobble control. The probable effort necessary to provide Skylab with active wobble damping raises the basic question as to whether or not CMG wobble damping is really necessary for a Skylab artificial gravity experiment. Skylab spinning about its axis of maximum moment of inertia has a large angular momentum of the order of a million ft-lb-sec and consequently is a very stable configuration. Only very large external torques can produce wobble of sufficient magnitude to be detrimental to crew comfort.

Although sufficient experimental work likely has not been conducted to firmly establish limits on man's physiological tolerances and adaptability to rotations, tentative limits have been suggested[4]. These indicate a ratio of spacecraft spin speed ω over wobble disturbance ω_d of

$$\omega/\omega_d \geq 50$$

for crew tolerance during normal operations. For limited time periods

$$\omega/\omega_d \geq 5$$

is considered acceptable.

For the worst case crew motion involving instantaneous movement of three astronauts from the Skylab mass center to the farthest extreme $\omega/\omega_d \approx 70$, which is within crew tolerance limits. Since in practice disturbances resulting from crew motions will be substantially less, individual crew movements would not necessitate active CMG wobble damping. Also, structural damping and dissipative effects of fluids carried within Skylab may be sufficient to prevent a long term cumulative buildup of wobble resulting from numerous crew disturbances. In addition, for the Skylab configuration modified with booms, significant damping might be obtained through a suitable design of the booms.

Other sources of Skylab wobble are venting torques, spin-up and spin-down, and environmental torques. By setting $K_1=K_2=0$ in the results of Appendix B, venting torques can be shown to produce negligible wobble for Skylab uncontrolled motion.

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4. Larson, C. A., "Rotating Space Station Stabilization Criteria for Artificial Gravity", NASA Technical Note TN D-5426, October 1969.

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APPENDIX A

WOBBLE DAMPING USING A MOMENTUM EXCHANGE DEVICE

SYSTEM DESCRIPTION

The configuration to be analyzed is shown in Fig.(7) and consists of a spinning satellite S carrying a system of bodies. The motion of these bodies relative to S can be prescribed so as to exchange momentum with S and thereby produce control torques on the satellite. (In practice these bodies are typically control moment gyros or momentum wheels but it is not necessary to specify these to formulate the problem.)

The mass center of S is termed S*. Principal axes of inertia of S at S* are designated X_1, X_2, X_3 and unit vectors $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$ are parallel to X_1, X_2, X_3, respectively. Corresponding principal moments of inertia are I_1, I_2, I_3, I_3 being the maximum moment of inertia. Unit vectors $\underline{i}_1'$, $\underline{i}_2'$, $\underline{i}_3'$ are parallel to a set of axes fixed in S which are slightly misaligned from X_1, X_2, X_3 and μ_1,μ_2,μ_3 is the 1,2,3 sequence of three axis Euler angles that describe the orientation of $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$ with respect to $\underline{i}_1'$, $\underline{i}_2'$, $\underline{i}_3'$.

The system of bodies carried within S is termed the controller C and the motion of that system relative to S is assumed prescribed so as to produce the control torque $\underline{T}^c$ on S

$$\underline{T}^c = - K_1 \omega_1' \underline{i}_1' - K_2 \omega_2' \underline{i}_2' \tag{1}$$

where K_1 and K_2 are positive constants and ω_1', ω_2', ω_3' are components of the inertial angular velocity of S in the directions of $\underline{i}_1'$, $\underline{i}_2'$, $\underline{i}_3'$, respectively.

EQUATIONS OF MOTION

It is convenient to write $\underline{T}^c$ in terms of components in the directions of principal axes. Since principal axes of S are only slightly misaligned from control axes, the transformation matrix relating unit vectors may be linearized

$$\begin{array}{c|ccc} & \underline{i}_1' & \underline{i}_2' & \underline{i}_3' \\ \hline \underline{i}_1 & 1 & \mu_3 & -\mu_2 \\ \underline{i}_2 & -\mu_3 & 1 & \mu_1 \\ \underline{i}_3 & \mu_2 & -\mu_1 & 1 \end{array} \qquad (2)$$

The inertial angular velocity ${}^I\underline{\omega}^S$ of S may be written

$${}^I\underline{\omega}^S = \omega_1' \, \underline{i}_1' + \omega_2' \, \underline{i}_2' + \omega_3' \, \underline{i}_3' = \omega_1 \, \underline{i}_1 + \omega_2 \, \underline{i}_2 + \omega_3 \, \underline{i}_3 \qquad (3)$$

where ω_1, ω_2, ω_3 are components in the directions of principal axes. Using Eq.(2) to eliminate $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$ gives

$$\begin{aligned} \omega_1' &= \omega_1 - \omega_2 \, \mu_3 + \omega_3 \, \mu_2 \\ \omega_2' &= \omega_2 + \omega_1 \, \mu_3 - \omega_3 \, \mu_1 \\ \omega_3' &= \omega_3 - \omega_1 \, \mu_2 + \omega_2 \, \mu_1 \end{aligned} \qquad (4)$$

Also, $\underline{T}^c$ may be expressed in terms of components in the directions of $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$

$$\underline{T}^c = -T_1^c \, \underline{i}_1 - T_2^c \, \underline{i}_2 - T_3^c \, \underline{i}_3 \qquad (5)$$

where*

---

*Numbers beneath equal signs are intended to direct attention to equations numbered correspondingly.

$$T_1^C \underset{(1,2)}{=} K_1\, \omega_1' + K_2\, \omega_2'\, \mu_3 \underset{(4)}{=} K_1(\omega_1-\omega_2\, \mu_3+\omega_3\, \mu_2) + K_2\, \omega_2\, \mu_3$$

$$T_2^C \underset{(1,2)}{=} -K_1\, \omega_1'\, \mu_3 + K_2\, \omega_2' \underset{(4)}{=} -K_1\, \omega_1\, \mu_3 + K_2(\omega_2+\omega_1\, \mu_3-\omega_3\, \mu_1)$$

$$T_3^C \underset{(1,2)}{=} K_1\, \omega_1'\, \mu_2 - K_2\, \omega_2'\, \mu_1 \underset{(4)}{=} K_1\, \omega_1\, \mu_2 - K_2\, \omega_2\, \mu_1 \tag{6}$$

and where higher order terms in μ_i, $i = 1,2,3$ have been dropped.

External torques will be neglected. Then, if $\underline{H}^S$ and $\underline{H}^C$ designate the angular momentum of the satellite S and the vector composite angular momentum of the bodies that make up the controller C, the equations of motion may be written

$$\frac{d\underline{H}^S}{dt} + \frac{d\underline{H}^C}{dt} = 0 \tag{7}$$

Since the motion of C is prescribed to produce on S the control torque of Eq.(1), then

$$\frac{d\underline{H}^S}{dt} = \underline{T}^C \tag{8}$$

and consequently the equations of motion of the controller are

$$\frac{d\underline{H}^C}{dt} = -\underline{T}^C \tag{9}$$

Expression of the angular momentum of S in the principal axis system, differentiation, and substitution of $\underline{T}^C$ from Eqs.(5) and (6) into Eq.(8) yields equations of motion for the satellite

$$I_1\, \dot{\omega}_1 + (I_3-I_2)\omega_3\, \omega_2 = -[K_1(\omega_1-\omega_2\, \mu_3+\omega_3\, \mu_2) + K_2\, \omega_2\, \mu_3] \tag{10}$$

$$I_2 \dot{\omega}_2 - (I_3-I_1)\omega_3\omega_1 = -[-K_1 \omega_1 \mu_3 + K_2(\omega_2+\omega_1 \mu_3-\omega_3 \mu_1)] \tag{11}$$

$$I_3 \dot{\omega}_3 - (I_1-I_2)\omega_1\omega_2 = - K_1 \omega_1 \mu_2 + K_2 \omega_2 \mu_1 \tag{12}$$

SOLUTION OF EQUATIONS OF MOTION

For Satellite

Eq.(12) may be written

$$\dot{\omega}_3 = \frac{1}{I_3} [(I_1-I_2)\omega_1 \omega_2 - K_1 \omega_1 \mu_2 + K_2 \omega_2 \mu_1] \tag{13}$$

Since the terms on the right-hand side involve a nonlinear term in ω_1 and ω_2 and ω_1 and ω_2 multiplied by μ_2 and μ_1, these terms may be considered small. Consequently, the time rate of change of ω_3 is small and ω_3 will be regarded as constant

$$\omega_3 = \omega_3\Big|_{t=0} = \omega \tag{14}$$

in the solving of Eqs.(10) and (11) for ω_1 and ω_2. The result will be considered valid during such time interval that

$$\omega >> \Delta\omega_3 \tag{15}$$

where $\Delta\omega_3$ is computed by substitution of the solutions for ω_1 and ω_2 into

$$\Delta\omega_3 = \frac{1}{I_3} \int_0^t [(I_1-I_2)\omega_1 \omega_2 - K_1 \omega_1 \mu_2 + K_2 \omega_2 \mu_1]d\xi \tag{16}$$

Replacing ω_3 by ω in Eqs.(10) and (11) and defining

$$\tau = \omega t \qquad d_1 = [(I_3-I_2)\omega + \mu_3(K_2-K_1)]/I_1\omega \qquad k_1 = K_1/I_1\omega$$

$$d_2 = [(I_3-I_1)\omega + \mu_3(K_1-K_2)]/I_2\omega \qquad k_2 = K_2/I_2\omega \tag{17}$$

gives

$$\frac{d\omega_1}{d\tau} + k_1\,\omega_1 + d_1\,\omega_2 = -k_1\omega\,\mu_2$$

$$\frac{d\omega_2}{d\tau} - d_2\,\omega_1 + k_2\,\omega_2 = k_2\omega\,\mu_1 \tag{18}$$

which has solutions

$$\omega_1 = V_1 + (A_1 \cos s_1\tau + A_2 \sin s_1\tau)e^{-s_0\tau}$$

$$\omega_2 = V_2 + (B_1 \cos s_1\tau + B_2 \sin s_1\tau)e^{-s_0\tau} \tag{19}$$

where

$$V_1 = -\frac{\omega k_2}{e}(\mu_1 d_1+\mu_2 k_1) \qquad V_2 = \frac{\omega k_1}{e}(\mu_1 k_2-\mu_2 d_2) \qquad e = d_1 d_2 + k_1 k_2 \tag{20}$$

and

$$s_0 = (k_1+k_2)/2 \qquad s_1 = (e-s_0^2)^{1/2}$$

$$A_1 = \omega_{10} - V_1 \qquad B_1 = \omega_{20} - V_2 \qquad A_2 = -\frac{1}{\delta_1}(B_1+A_1\delta_0)$$

$$B_2 = \frac{1}{\delta_1}[A_1(\delta_0^2 + \delta_1^2) + B_1\,\delta_0] \tag{21}$$

for

$$\omega_{i0} = \omega_i \Big|_{\tau=t=0}, \qquad i=1,2$$

$$\delta_0 = (k_1-s_0)/d_1 \qquad \delta_1 = s_1/d_1 \tag{22}$$

Also, from Eqs.(4), (14) and (19)

$$\omega_1' = V_1' + [(A_1-B_1\mu_3)\cos s_1\tau + (A_2-B_2\mu_3)\sin s_1\tau]e^{-s_0\tau}$$

$$\omega_2' = V_2' + [(B_1+A_1\mu_3)\cos s_1\tau + (B_2+A_2\mu_3)\sin s_1\tau]e^{-s_0\tau} \tag{23}$$

where

$$V_1' = V_1 + \omega\mu_2 \underset{(20)}{=} \frac{\omega d_1}{e}(-\mu_1 k_2+\mu_2 d_2)$$

$$V_2' = V_2 - \omega\mu_1 \underset{(20)}{=} -\frac{\omega d_2}{e}(\mu_1 d_1+\mu_2 k_1) \tag{24}$$

Since Eqs.(19) and (23) show that satellite angular velocities decay to constant steady state values, the prescribed control is seen to eliminate wobble.

The $A_i\mu_3$ and $B_i\mu_3$ terms in Eq.(23) are small in comparison to A_i and B_i, i=1,2, and can be neglected. Also, for I_3-I_2 or I_3-I_1 not nearly zero or for K_1 and K_2 not so large that control torques dominate satellite inertia torques (a likely prospect for a momentum exchange controller of practical size), the terms proportional to μ_3 in the definitions (Eq.(17)) for d_1 and d_2 are negligible in Eqs.(19,20; 23,24). Consequently, for these cases, small μ_3 misalignments do not significantly affect satellite motion. However, for small I_3-I_2 or I_3-I_1 or large K_1 or K_2, μ_3 terms in d_1 and d_2 must be retained.

Now an estimate of the deviation $\Delta\omega_3$ of ω_3 from its assumed constant value will be obtained by substituting

ω_1 and ω_2 from Eq.(19) into Eq.(16) and integrating to yield

$$\frac{\Delta\omega_3}{\omega} = \zeta_1\tau + \zeta_2 + e^{-s_0\tau}(\zeta_3 \sin s_1\tau + \zeta_4 \cos s_1\tau)$$

$$+ e^{-2s_0\tau}(\zeta_5 \sin 2s_1\tau + \zeta_6 \cos 2s_1\tau) \qquad (25)$$

where $\zeta_2,\ldots,\zeta_6$ are proportional to $\mu_i\mu_j$, $\frac{\omega_{i0}}{\omega}\frac{\omega_{j0}}{\omega}$, and $\frac{\omega_{i0}}{\omega}\mu_j$. For initial values ω_{10} and ω_{20} of ω_1 and ω_2 very small in comparison with initial value ω of ω_3, the terms involving $\zeta_2,\ldots,\zeta_6$ are very small. The value of the $\zeta_1\tau$ term at decay time τ_D when the envelopes of the wobble angular velocities have decayed to 10% of initial values is determined from

$$e^{-s_0\tau_D} \underset{(21)}{=} e^{-\left(\frac{k_1+k_2}{2}\right)\tau_D} = 0.1 \qquad (26)$$

Solution for τ_D gives

$$\tau_D = \frac{4.6}{k_1+k_2} \qquad (27)$$

and $\zeta_1\tau_D$ can be shown to be a very small quantity proportional to quadratic terms in μ_i, i=1,2 which goes to zero for k_1 and k_2 approaching zero and goes to a maximum of

$$\frac{\Delta\omega_3}{\omega} \doteq \zeta_1\tau_D \rightarrow \frac{4.6}{(k_1+k_2)}\left(\mu_1^2\frac{I_2}{I_3}k_2 + \mu_2^2\frac{I_1}{I_3}k_1\right) \qquad (28)$$

for k_1 and k_2 approaching infinity. Consequently, angular velocities determined from Eqs.(19) and (23) are accurate not only throughout the interval $0\leq\tau\leq\tau_D$ in which wobble rates are significant but also for τ up to many times τ_D.

For Controller

It is convenient to express equations of motion Eq.(9) for C in an inertially fixed coordinate system with axes parallel to initial directions of satellite principal axes. First, Euler angles relating orientation of satellite principal axes with respect to their initial directions must be determined.

Let a 3,2,1 sequence of three axes Euler angles ϕ_3, ϕ_2, ϕ_1 describe the orientation of $\underline{i}_1$, $\underline{i}_2$, $\underline{i}_3$ with respect to inertially fixed unit vectors $\underline{i}_{10}$, $\underline{i}_{20}$, $\underline{i}_{30}$ where $\underline{i}_{\ell 0}=\underline{i}_\ell\big|_{t=0}$, $\ell=1,2,3$. Eq.(19) for angular velocities suggest that the satellite axis of maximum moment of inertia X_3 experiences small motions in the neighborhood of its initial direction so that ϕ_1 and ϕ_2 may be considered to have small values. Then the linearized transformation matrix between unit vectors is written

$$\begin{array}{cccc} & \underline{i}_{10} & \underline{i}_{20} & \underline{i}_{30} \\ \underline{i}_1 & c\phi_3 & s\phi_3 & -\phi_2 \\ \underline{i}_2 & -s\phi_3 & c\phi_3 & \phi_1 \\ \underline{i}_3 & \phi_1\, s\phi_3 + \phi_2\, c\phi_3 & \phi_2\, s\phi_3 - \phi_1\, c\phi_3 & 1 \end{array} \qquad (29)$$

where $\quad s\phi_3 = \sin\phi_3\ , \quad c\phi_3 = \cos\phi_3$

Also,

$$I_{\underline{\omega}}^{S} = \omega_1\,\underline{i}_1 + \omega_2\,\underline{i}_2 + \omega_3\,\underline{i}_3 = (\dot\phi_1-\dot\phi_3\phi_2)\underline{i}_1 + (\dot\phi_2+\dot\phi_3\phi_1)\underline{i}_2 + \dot\phi_3\,\underline{i}_3 \qquad (30)$$

Comparison of terms gives

$$\omega_3 = \dot\phi_3 \underset{(14)}{=} \omega \qquad (31)$$

and $\quad \omega_1 \underset{(31)}{=} \dot\phi_1 - \omega\phi_2 \qquad \omega_2 \underset{(31)}{=} \dot\phi_2 + \omega\phi_1 \qquad (32)$

Using Eq.(17) to write Eq.(32) in terms of τ, substituting ω_1 and ω_2 from Eq.(19) and solving the results yield

$$\phi_i = \phi_{i1} + \phi_{i2} \sin\tau + \phi_{i3} \cos\tau + e^{-s_0\tau} (\phi_{i4}\sin s_1\tau + \phi_{i5}\cos s_1\tau)$$

$$i=1,2 \qquad (33)$$

where
$$\phi_{11} = \frac{V_2}{\omega} \qquad \phi_{21} = -\frac{V_1}{\omega} \qquad (34)$$

and ϕ_{ij}, i=1,2; j=2,...,5 involve terms proportional to V_ℓ and $\omega_{\ell 0}$, ℓ=1,2.

Now Eq.(9) can be resolved into the inertially fixed axes system parallel to $\underline{i}_{10}$, $\underline{i}_{20}$, $\underline{i}_{30}$

$$\frac{dH_1}{dt} \underset{(5,29)}{=} T_1^C\, c\phi_3 - T_2^C\, s\phi_3 + T_3^C(\phi_1 s\phi_3 + \phi_2 c\phi_3)$$

$$\frac{dH_2}{dt} \underset{(5,29)}{=} T_1^C\, s\phi_3 + T_2^C\, c\phi_3 + T_3^C(-\phi_1 c\phi_3 + \phi_2 s\phi_3)$$

$$\frac{dH_3}{dt} \underset{(5,29)}{=} -T_1^C\, \phi_2 + T_2^C\, \phi_1 + T_3^C \qquad (35)$$

where H_1, H_2, H_3 are components of the controller angular momentum in the inertial reference frame.

Integrating both sides of the first two of Eq.(35), using Eq.(6) and Eq.(31), and dropping $\phi_i\mu_j$ terms, i=1,2;

j=1,2,3 gives

$$H_1 = H_1|_{\tau=0} + \frac{1}{\omega}\int_0^\tau (K_1\omega_1' \cos\xi - K_2\omega_2' \sin\xi)d\xi$$

$$+ \frac{\mu_3}{\omega}\int_0^\tau (K_1\omega_1' \sin\xi + K_2\omega_2' \cos\xi)d\xi \qquad (36)$$

$$H_2 = H_2|_{\tau=0} + \frac{1}{\omega}\int_0^\tau (K_1\omega_1' \sin\xi + K_2\omega_2' \cos\xi)d\xi$$

$$- \frac{\mu_3}{\omega}\int_0 (K_1\omega_1' \cos\xi - K_2\omega_2' \sin\xi)d\xi \qquad (37)$$

The second integrals of Eqs.(36) and (37) can be dropped in comparison to the first and evaluation of the remaining integrals yields the changes $\Delta H_i = H_i(\tau) - H_i|_{\tau=0}$, i=1,2 in the components of angular momentum of C.

$$\Delta H_1 = \frac{1}{\omega}[K_1V_1' \sin\tau + K_2V_2'(\cos\tau-1)] + N_3 - N_4$$

$$+ e^{-s_0\tau}(N_1 \sin U_1\tau + N_2 \sin U_2\tau - N_3 \cos U_1\tau + N_4 \cos U_2\tau)$$

$$\Delta H_2 = \frac{1}{\omega}[-K_1V_1'(\cos\tau-1) + K_2V_2' \sin\tau] + N_1 + N_2$$

$$+ e^{-s_0\tau}(-N_3 \sin U_1\tau + N_4 \sin U_2\tau - N_1 \cos U_1\tau - N_2 \cos U_2\tau)$$

(38)

where

$$U_1 = 1 + s_1 \qquad U_2 = 1 - s_1$$

$$N_1 = (-P_1 s_0 + P_4 U_1)/(s_0^2 + U_1^2) \qquad N_2 = (P_2 s_0 - P_3 U_2)/(s_0^2 + U_2^2)$$

$$N_3 = (P_1 U_1 + P_4 s_0)/(s_0^2 + U_1^2) \qquad N_4 = (P_3 s_0 + P_2 U_2)/(s_0^2 + U_2^2) \tag{39}$$

and

$$P_1 = (K_1 A_2 - K_2 B_1)/2\omega \qquad P_2 = (K_1 A_2 + K_2 B_1)/2\omega$$

$$P_3 = (-K_1 A_1 + K_2 B_2)/2\omega \qquad P_4 = (K_1 A_1 + K_2 B_2)/2\omega \tag{40}$$

Now, the steady state change in angular momentum of the controller is of primary concern. Eq.(38) shows a periodic change in an inertially fixed plane nearly perpendicular to the satellite axis of maximum moment of inertia. The maximum magnitude ΔH_{max} of this periodic change can be shown to be

$$\begin{aligned}\Delta H_{max} &= \max\left(\Delta H_1^2 + \Delta H_2^2\right)^{1/2} \\ &= \left[\left(N_3 - N_4 - \frac{K_2 V_2'}{\omega}\right)^2 + \left(N_1 + N_2 + \frac{K_1 V_1'}{\omega}\right)^2\right]^{1/2} + \left[\left(\frac{K_1 V_1'}{\omega}\right)^2 + \left(\frac{K_2 V_2'}{\omega}\right)^2\right]^{1/2}\end{aligned} \tag{41}$$

Integration of the third of Eq.(35) reveals ΔH_3 involves a secular term, sinusoidal terms, and decaying sinusoidal terms, all of which are proportional to second and higher terms in μ_i and ω_{i0}/ω. Since all these except the time growing term are small in comparison to terms of ΔH_1 and ΔH_2, only the secular term will be carried in the description for ΔH_3.

$$\Delta H_3 = \frac{1}{\omega^2}[K_1 V_1'^2 + K_2 V_2'^2 - K_1 V_1 V_1' d_2 - K_2 V_2 V_2' d_1 + V_1' V_2' (K_1 k_2 - K_2 k_1)]\tau \tag{42}$$

APPENDIX B

WOBBLE DAMPING FOR SMALL, CONSTANT, BODY-FIXED TORQUES

The effects of small, constant, body-fixed torques on satellite and controller motion is investigated here. For control torques

$$\underline{T}^C = -K_1 \omega_1 \underline{i}_1 - K_2 \omega_2 \underline{i}_2 \tag{43}$$

where $\underline{i}_1$, $\underline{i}_2$, ω_1, ω_2 are in the directions of satellite principal axes and for a constant external torque $\underline{T}^E$

$$\underline{T}^E = T_1 \underline{i}_1 + T_2 \underline{i}_2 + T_3 \underline{i}_3 \tag{44}$$

satellite equations of motion are

$$I_1 \dot{\omega}_1 + (I_3 - I_2)\omega_3 \omega_2 = T_1 - K_1 \omega_1 \tag{45}$$

$$I_2 \dot{\omega}_2 - (I_3 - I_1)\omega_3 \omega_1 = T_2 - K_2 \omega_2 \tag{46}$$

$$I_3 \dot{\omega}_3 - (I_1 - I_2)\omega_1 \omega_2 = T_3 \tag{47}$$

Eq.(47) may be rewritten

$$\dot{\omega}_3 = \frac{1}{I_3}[(I_1 - I_2)\omega_1 \omega_2 + T_3] \tag{48}$$

Since the first term on the right-hand side is nonlinear it may be considered small and if attention is restricted to very small external torques, then T_3/I_3 is also a small quantity. Consequently, the rate of change of ω_3 is a very small quantity and ω_3 will be regarded as constant

$$\omega_3 = \omega_3\big|_{t=0} = \omega \tag{49}$$

in Eqs.(45) and (46). The resulting solutions will be considered accurate during such time interval for which $\omega >> \Delta\omega_3$ where $\Delta\omega_3$ is computed by placing the solutions for ω_1 and ω_2 into

$$\Delta\omega_3 = \frac{1}{I_3} \int_0^t [(I_1-I_2)\omega_1 \omega_2 + T_3]d\xi \tag{50}$$

Defining

$$b_1 = \frac{I_3-I_2}{I_1} \qquad b_2 = \frac{I_3-I_1}{I_2} \qquad L_1 = T_1/\omega I_1 \qquad L_2 = T_2/\omega I_2 \tag{51}$$

and τ, k_1, and k_2 as in Eq.(17) of Appendix A, Eqs.(45) and (46) may be rewritten

$$\frac{d\omega_1}{d\tau} + k_1 \omega_1 + b_1 \omega_2 = L_1$$

$$\frac{d\omega_2}{d\tau} - b_2 \omega_1 + k_2 \omega_2 = L_2 \tag{52}$$

These equations of motion for the satellite together with the equations of motion for the controller are the same in character as Eqs.(18) and (9) and have solutions of the same form. Solution of Eq.(52) yields

$$\omega_1 = W_1 + (D_1 \cos s_1\tau + D_2 \sin s_1\tau)e^{-s_0\tau}$$

$$\omega_2 = W_2 + (E_1 \cos s_1\tau + E_2 \sin s_1\tau)e^{-s_0\tau} \tag{53}$$

where

$$W_1 = (L_1k_2 - L_2b_1)/f \quad W_2 = (L_1b_2 + L_2k_1)/f \quad f = b_1b_2 + k_1k_2$$

$$D_1 = \omega_{10} - W_1 \qquad E_1 = \omega_{20} - W_2 \qquad D_2 = -\frac{1}{\delta_1}(E_1 + D_1\ \delta_0)$$

$$E_2 = \frac{1}{\delta_1}[D_1(\delta_0^2 + \delta_1^2) + E_1\ \delta_0] \tag{54}$$

with ω_{10}, ω_{20}, δ_0, δ_1, s_0, s_1 defined as in Eqs.(21) and (22) of Appendix A except for d_1 and d_2 replaced by b_1 and b_2.

Also, the value ΔH_{max} of the maximum magnitude of the controller periodic component of accumulated angular momentum in a plane nominally perpendicular to X_3 and the value ΔH_3 of the time increasing component nominally in the direction of X_3 are given by relations similar to Eqs.(41) and (42) of Appendix A.

$$\Delta H_{max} = \left[\left(M_3 - M_4 - \frac{K_2W_2}{\omega}\right)^2 + \left(M_1 + M_2 + \frac{K_1W_1}{\omega}\right)^2\right]^{1/2}$$

$$+ \left[\left(\frac{K_1W_1}{\omega}\right)^2 + \left(\frac{K_2W_2}{\omega}\right)^2\right]^{1/2} \tag{55}$$

$$\Delta H_3 = \frac{1}{\omega^2}[K_1W_1^2(1-b_2) + K_2W_2^2(1-b_1) + W_1W_2(K_1k_2-K_2k_1)]\tau \tag{56}$$

where M_i is computed just as N_i, i=1,...,4 of Eq.(39) of Appendix A except A_1, B_j are replaced by D_j, E_j, j=1,2, respectively.

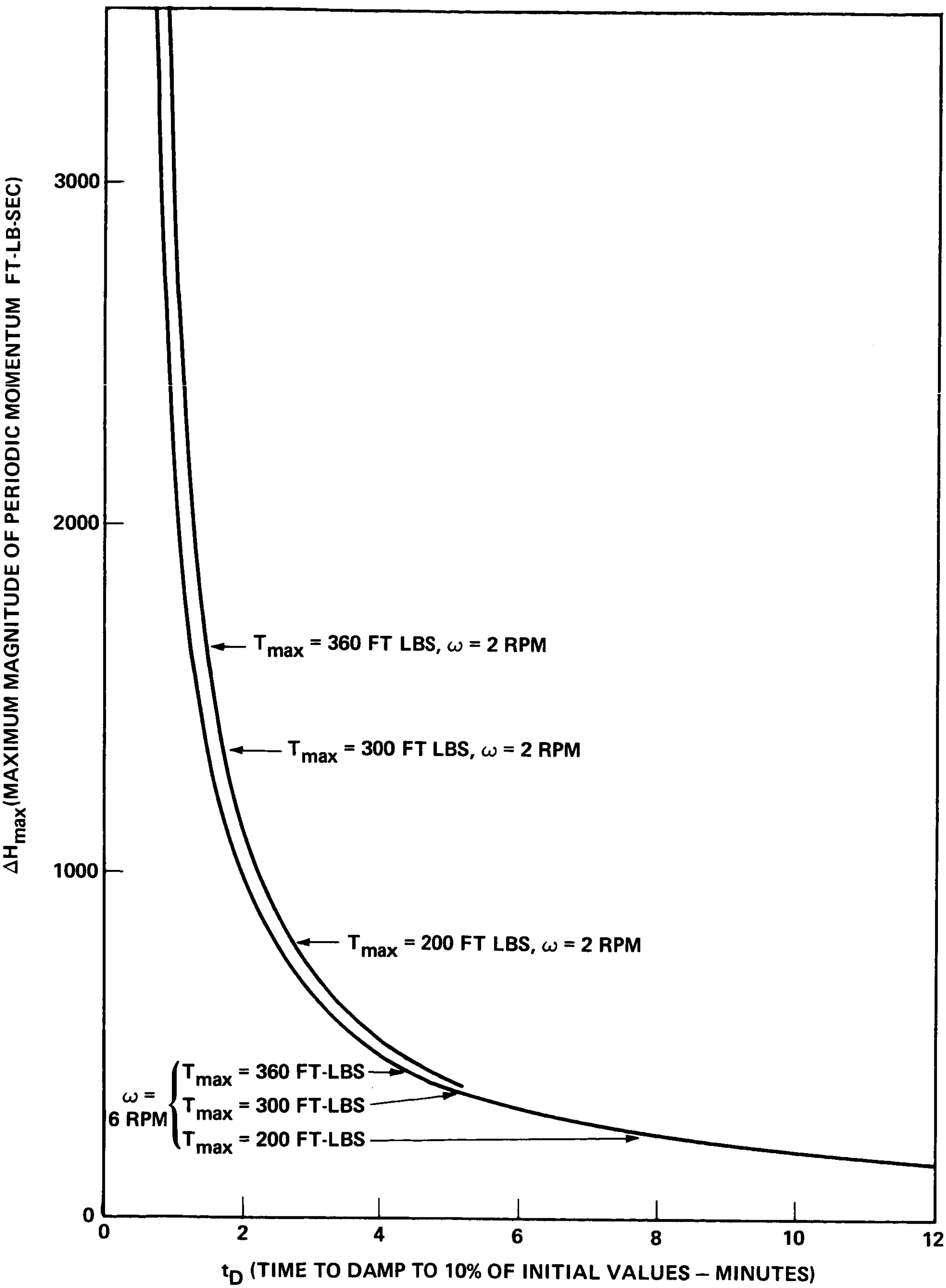


FIGURE 1 - MAXIMUM MAGNITUDE OF PERIODIC COMPONENT OF CMG ANGULAR MOMENTUM VS. DAMPING TIME FOR ONE DEGREE PRINCIPAL AXIS MISALIGNMENT AND WORST CASE CREW MOTION

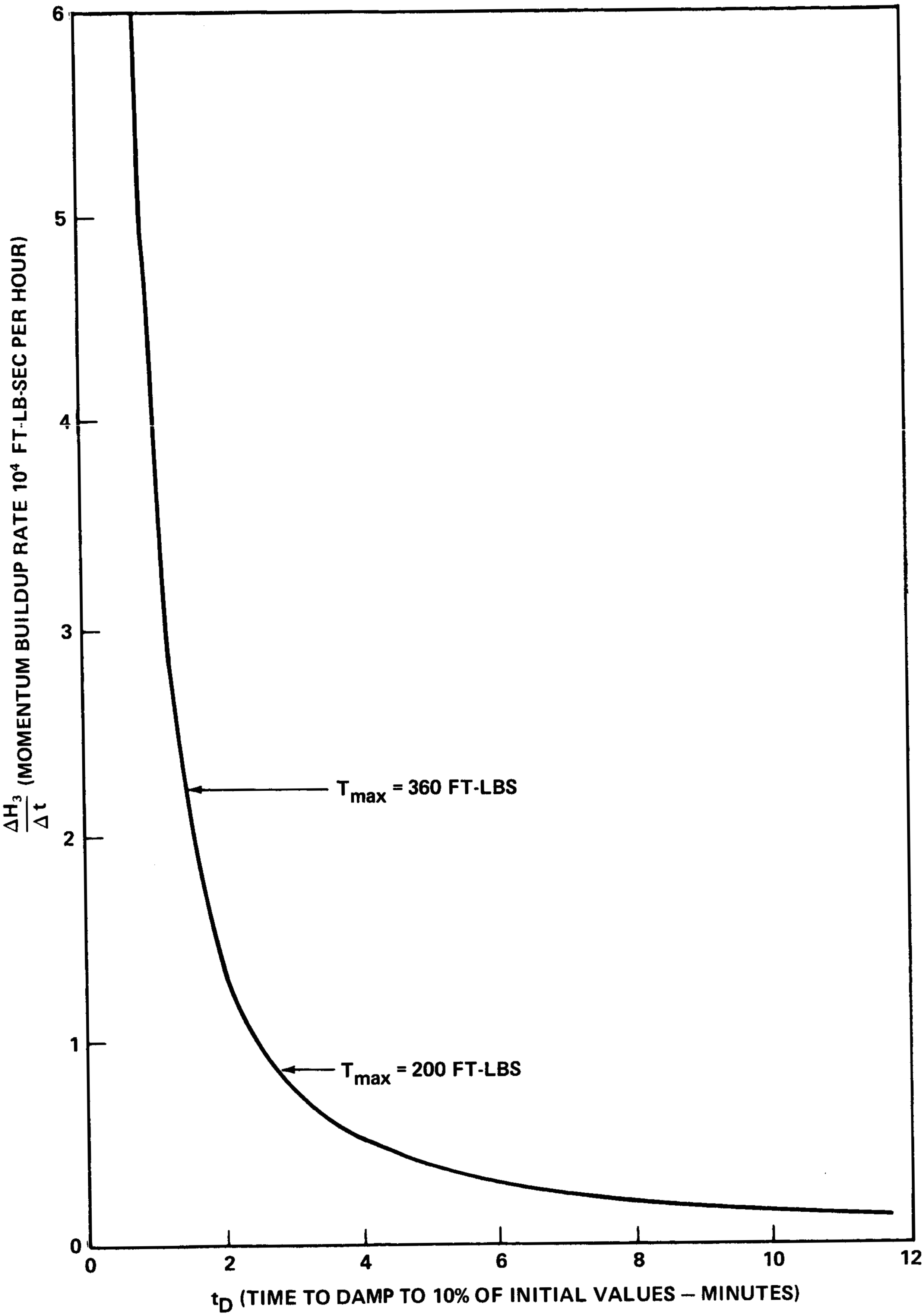


FIGURE 2 - BUILDUP RATE OF SECULAR COMPONENT OF CMG ANGULAR MOMENTUM VS. DAMPING TIME FOR ONE DEGREE PRINCIPAL AXIS MISALIGNMENT AND ω = 2 RPM

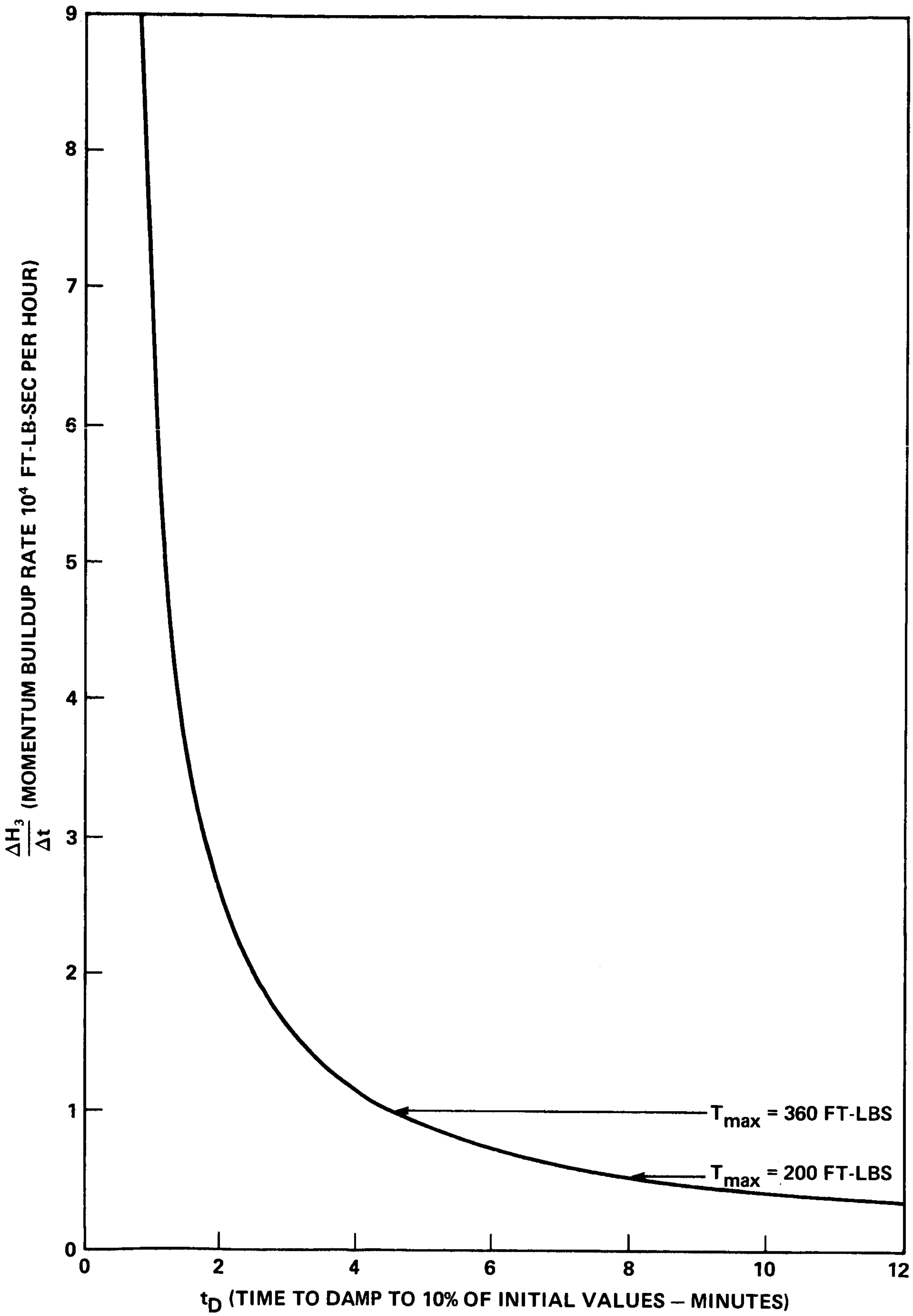


FIGURE 3 - BUILDUP RATE OF SECULAR COMPONENT OF CMG ANGULAR MOMENTUM VS. DAMPING TIME FOR ONE DEGREE PRINCIPAL AXIS MISALIGNMENT AND ω = 6 RPM

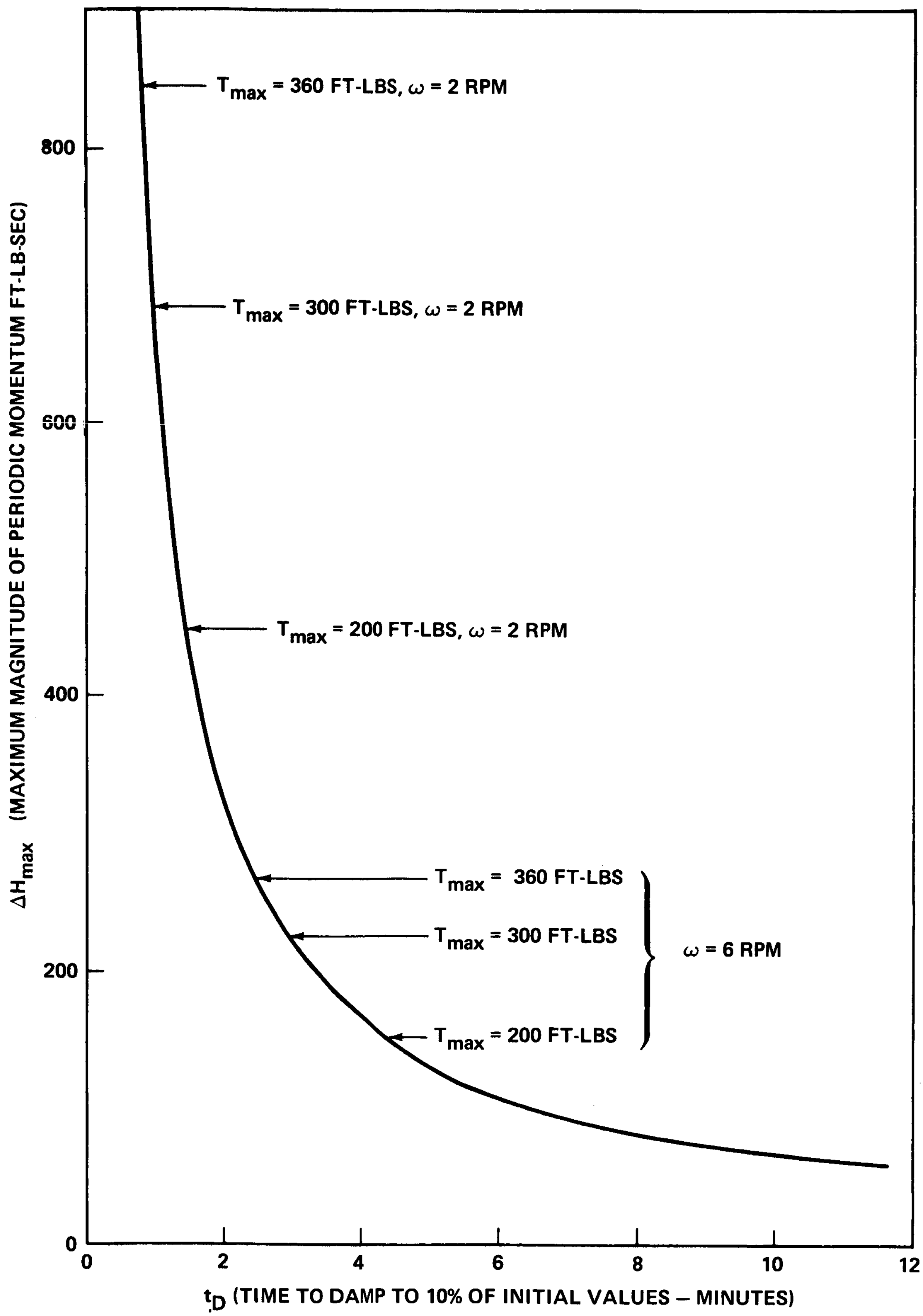


FIGURE 4 - MAXIMUM MAGNITUDE OF PERIODIC COMPONENT OF CMG ANGULAR MOMENTUM VS. DAMPING TIME FOR WORST CASE CREW MOTION

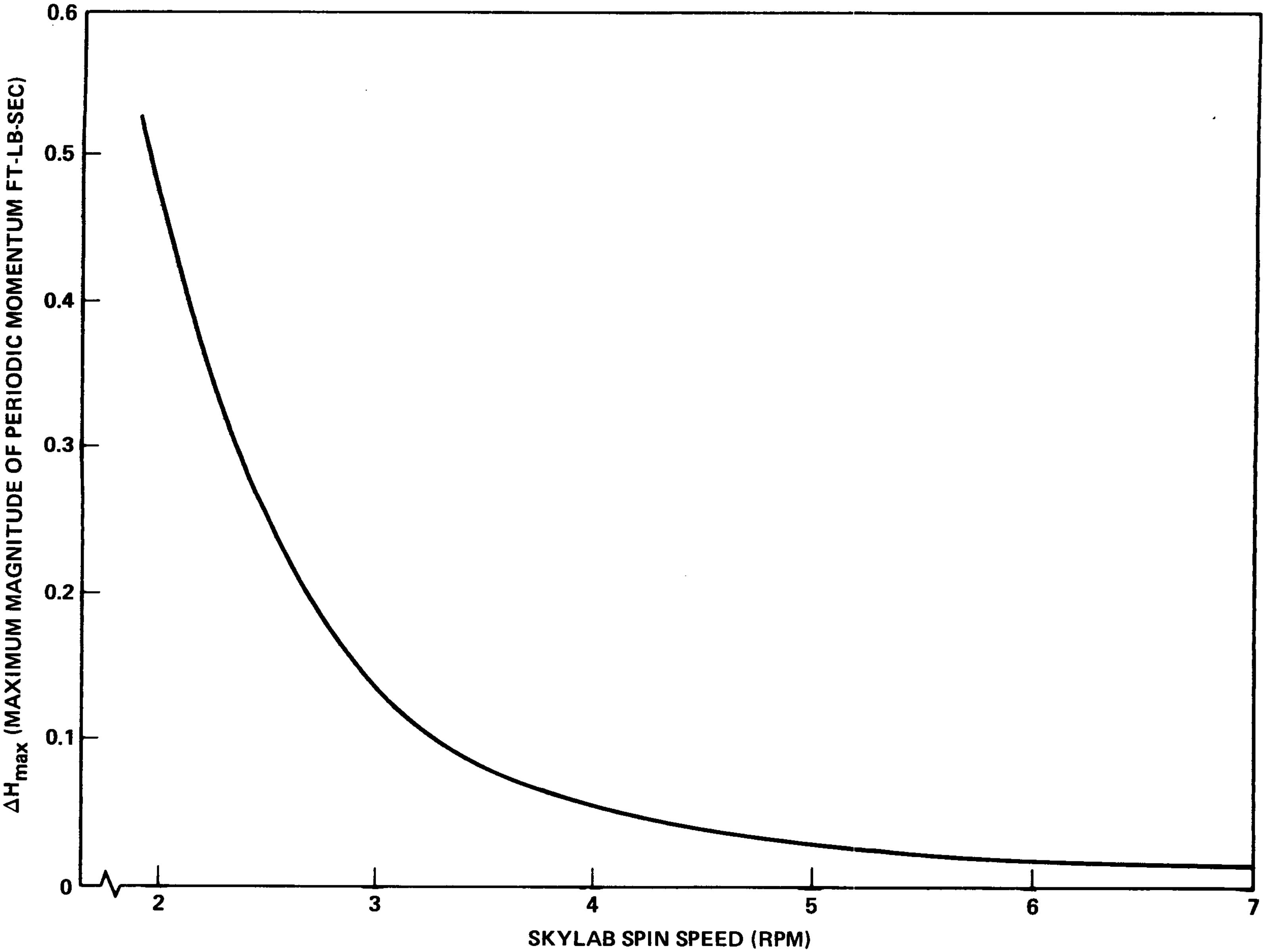


FIGURE 5 - MAXIMUM MAGNITUDE OF PERIODIC COMPONENT OF CMG ANGULAR MOMENTUM FOR SKYLAB MEAN VENTING TORQUES $T_x = 0.096$ N. M., $T_y = 0.048$ N. M., $T_z = 0.187$ N. M.

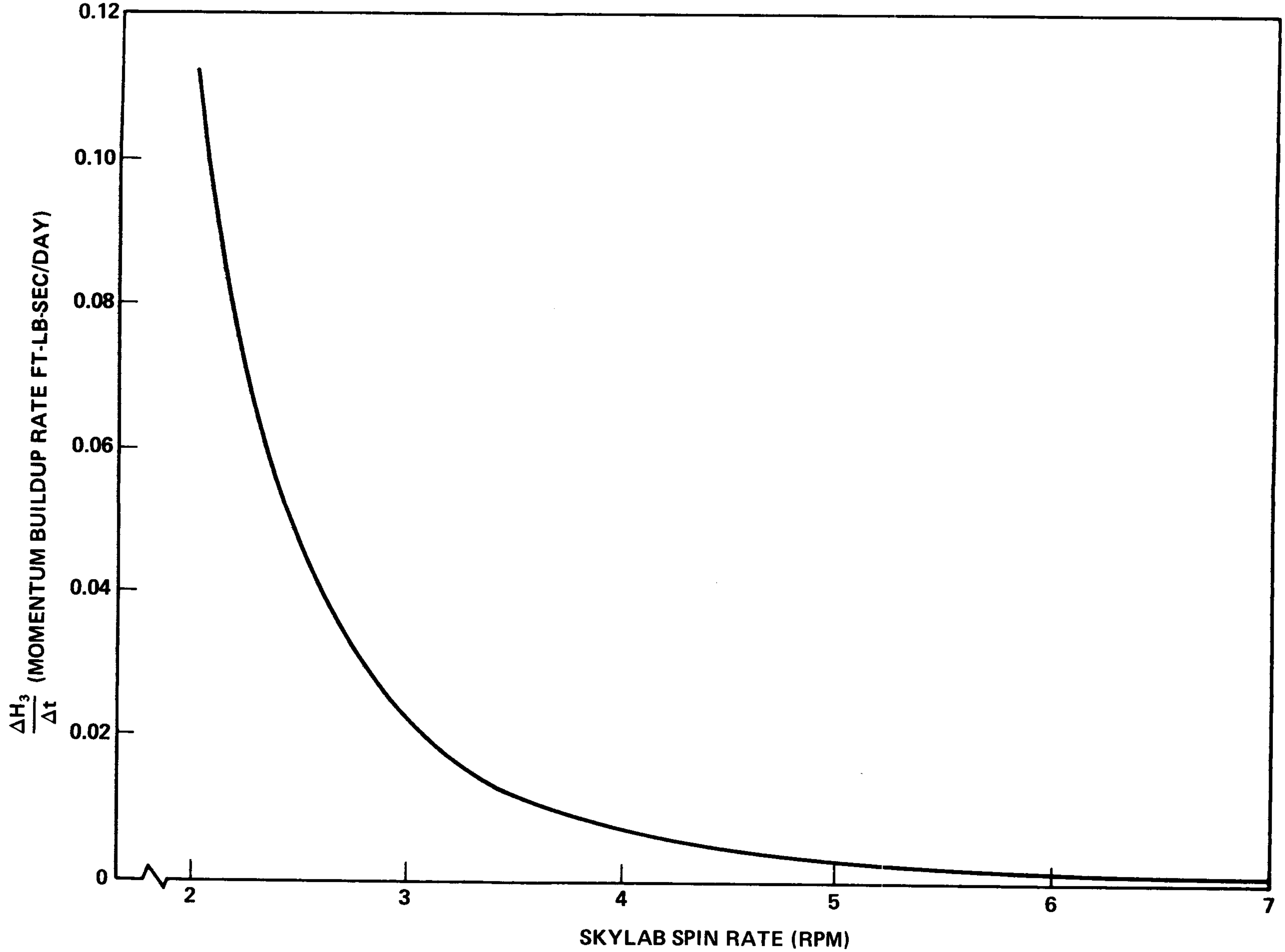
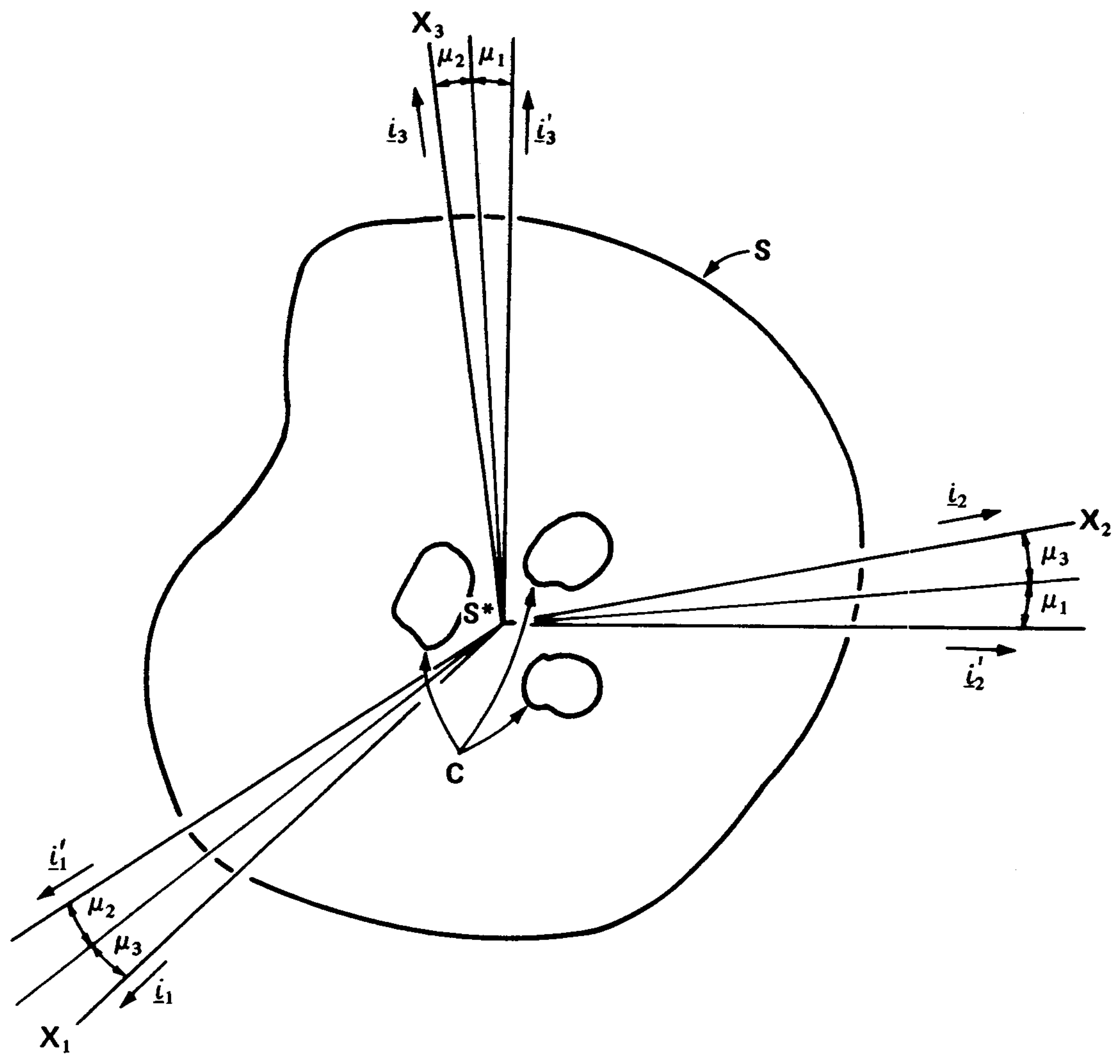


FIGURE 6 - BUILDUP RATE OF SECULAR COMPONENT OF CMG ANGULAR MOMENTUM FOR SKYLAB MEAN VENTING TORQUES T_x = 0.096 N. M., T_y = 0.048 N. M., T_z = 0.187 N. M.



X_3
μ_2
μ_1
$\underline{i}_3$
$\underline{i}_3'$
S
S*
$\underline{i}_2$
X_2
μ_3
μ_1
$\underline{i}_2'$
C
$\underline{i}_1'$
μ_2
μ_3
$\underline{i}_1$
X_1

FIGURE 7